MATLAB Modul 4

Introduction to Computational Science:
Modeling and Simulation for the Sciences, 2nd Edition
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Introduction

Modul 4 prepares you to use *MATLAB* for material in Chapter 9 and subsequent chapters.

Besides being a system with powerful commands, *MATLAB* is a programming language. In this tutorial, we consider some of the commands for the higher level constructs selection and looping that are available in *MATLAB*. Besides more traditional applications, looping enables us to generate graphical animations that help in visualization of concepts.

Random Numbers

Random numbers are essential for computer simulations of real-life events, such as weather or nuclear reactions. To pick the next weather or nuclear event, the computer generates a sequence of numbers, called **random numbers** or **pseudorandom numbers**. As we discuss in Module 9.2 , "Simulations," an algorithm actually produces the numbers; so they are not really random, but they appear to be random. A uniform random number generator produces numbers in a **uniform distribution** with each number having an equal likelihood of being anywhere within a specified range. For example, suppose we wish to generate a sequence of uniformly distributed, four-digit random integers. The algorithm used to accomplish this should, in the long run, produce approximately as many numbers between, say, 1000 and 2000 as it does between 8000 and 9000.

Definition **Pseudorandom numbers** (also called **random numbers**) are a sequence of numbers that an algorithm produces but which appear to be generated randomly. The sequence of random numbers is **uniformly distributed** if each random number has an equal likelihood of being anywhere within a specified range.

MATLAB provides the random number generator ***rand***. Each call to *rand* returns a uniformly distributed pseudorandom floating point number between 0 and 1. Evaluate the following command several times to observe the generation of different random numbers:

```
rand
```

For example, executing the command three times generates floating point output between 0 and 1, such as 0.296615, 0.908778, and 0.056155.

With one integer argument, ***rand*(*n*)** returns a square array with n rows and n columns of random numbers between 0 and 1. Similarly, ***rand*(*m*, *n*)** returns an array with m rows and n columns of random numbers.

Suppose, however, we need a random floating point number from 0.0 up to 5.0. Because the length of this interval is 5.0, we multiply by this value (5.0 * *rand*) to stretch the interval of numbers. Mathematically we have the following:

$$0.0 \le rand < 1.0$$

Thus, multiplying by 5.0 throughout, we obtain the correct interval, as shown:

$$0.0 \le 5.0 * rand < 5.0$$

If the lower bound of the range is different from 0, we must also add that value. For example, if we need a random floating point number from 2.0 up to 7.0, we multiply by the length of the interval, 7.0 – 2.0 = 5.0, to expand the range. We then add the lower bound, 2.0, to shift or translate the result.

$$2.0 \le (7.0 - 2.0) * rand + 2.0 < 7.0$$

Generalizing, we can obtain a **uniformly distributed floating point number from *a* up to *b*** with the following computation:

$$(b - a) * rand + a$$

Quick Review Question 1 Start a new *Script*. In opening comments, have "MATLAB Tutorial 4 Answers" and your name. Save the file under the name *MATLABTutorial4Ans.m.* In the file, preface this and all subsequent Quick Review Questions with a comment that has "%% QRQ" and the question number, such as follows:

```
%% QRQ 1 a
```

a. Generate a 3-by3 array of random floating point numbers between 0 and 1.
b. Generate a 1-by-5 array of random floating point numbers between 0 and 10.
c. Generate a 2-by-4 array of random floating point numbers between 8 and 12.

To obtain an integer random number, we use the ***randi*** function. The function call ***randi*(*imax*)** returns a random integer between 1 and *imax*, inclusively. Thus, the following command returns a random integer from the set {1, 2, …, 25}:

```
randi(25)
```

For arrays we employ ***randi*(*imax*, *n*)** and ***randi*(*imax*, *m*, *n*)** to return n-by-n and m-by-n arrays of random integers between 1 and *imax*, respectively. Thus, the following commands assign to *matSqr* a 4-by-4 array of integers from the set {1, 2, …, 25} and to *matRect* a 2-by-3 array of integers from the same set:

```
matSqr = randi(25, 4)

matRect = randi(25, 2, 3)
```

For an array of random integers between *imin* and *imax*, we can use the form ***randi*([*imin*, *imax*], *n*)** and ***randi*([*imin*, *imax*], *m*, *n*)** to return n-by-n and m-by-n arrays, respectively.

Thus, to return a 2-by-3 array of integers in the set {0, 1, 2, …, 24}, we can employ the following command:

```
randi([0, 24], 2, 3)
```

Quick Review Question 2

a. Give the command to generate a number representing a random throw of a die with a return value of 1, 2, 3, 4, 5, or 6.

b. Give the command to generate a random number representing ages from 18 to 22, inclusively, that is, a random integer from the set {18, 19, 20, 21, 22}.

A random number generator starts with a number, which we call a **seed** because all subsequent random numbers sprout from it. The generator uses the seed in a computation to produce a pseudorandom number. The algorithm employs that value as the seed in the computation of the next random number, and so on.

Typically, we seed the random number generator once at the beginning of a program. ***rand*('*seed', n*)** seeds the random number generator with the integer *n*. For example, we seed the random number generator with 14234 as follows:

```
rand('seed', 14234)
```

If the random number generator always starts with the same seed, it always produces the same sequence of numbers. A program using this generator performs the same steps with each execution. The ability to reproduce detected errors is useful when debugging a program.

However, this replication is not desirable when we are using the program. Once we have debugged a function that incorporates a random number generator, such as for a computer simulation, we want to generate different sequences each time we call the function. For example, if we have a computer simulation of weather, we do not want the program always to start with a thunderstorm. The following command seeds the random number generator using the time of day so that a different sequence of random numbers occurs for each run of a simulation:

```
rand('seed', sum(100*clock))
```

clock returns a six-element array with numeric information about the date and time, including the seconds as a 2-digit number. We multiply the numbers in the array by 100 and find their sum to use as a seed for the random number generator.

Quick Review Question 3

a. Write a command to generate a 10-by-10 array of random integers from 1 through 100, inclusively, that is, from the set {1, 2, 3, ..., 99, 100}. Using the up-arrow to repeat the command, execute the expression several times, and notice that the list changes each time.

b. Using the up-arrow, retrieve the command from Part a. Before the command, seed the random number generator with a four-digit number. Execute the two commands in sequence several times, and notice that the list does not change.

Quick Review Question 4 Using the form that appears on several lines, write an *if* statement to generate and test a random floating point number between 0 and 1. If the number is less than 0.3, return 1; otherwise, return 0.

Modulus Function

An algorithm for a random number generator often employs the modulus function, ***rem*** in *MATLAB*, which gives the remainder of a first integer argument divided by a second integer argument. To return the remainder of the division of *m* by *n*, we employ a command of the following form:

```
rem(m, n)
```

(This call is equivalent to *m* % *n* in C, C++, Java, and Python). Thus, the following statement returns 3, the remainder of 23 divided by 4.

```
rem(23, 4)
```

Quick Review Question 4 Assign 10 to *r*. Then, assign to *r* the remainder of the division of 7*r* by 11. Before executing the command, calculate the final value of *r* to check your work.

Counting

Frequently, we employ arrays in *MATLAB*; and instead of using a loop, we can use the function ***sum*** to count items in the list that match a pattern. As the segment below illustrates, *sum* provides an alternative to a *for* loop for counting the number of random numbers less than 0.3. First, we generate a table of 0s and 1s, such that if a random number is less than 0.3, the table entry is 1. Then we count the elements in the table that match the pattern 1.

```
tbl = rand(1, 10) < 0.3
sum(tbl)
```

Output of the table and the number of ones in the table might be as follows:

```
tbl =
       0   1   0   0   0   0   0   1   1   0
ans =
       3
```

Quick Review Question 6 Write a statement to generate a table of 20 random floating point numbers between 0 and 1 and store the result in variable *lst*. Give one

statement to return the number of these values less than 0.4 and another statement to return the number of values greater than or equal to 0.6.

Basic Statistics

The function ***mean*** in this package returns the **mean**, or average, of the elements in an array and has the following format:

```
mean(list)
```

Similarly, ***std*** returns the **standard deviation** of the elements in a list. The following segment creates a list of 10 floating point numbers and returns the mean and standard deviation:

```
tbl = rand(1, 10)
mean(tbl)
std(tbl)
```

Quick Review Question 7

a. Generate 10,000 **normally distributed random numbers** using the function ***randn*** with the same format as *rand*. Store the results in a vector, *normalNums*. Do not display *normalNums*.

b. Determine the mean of *normalNums*.

c. Determine the standard deviation of *normalNums*.

d. In a normal distribution, the mean is 0 and the standard deviation is 1. Are your answers from Parts b and c approximately equal to these values?

Histogram

A **histogram** of a data set is a bar chart where the base of each bar is an interval of data values and the height of this bar is the number of data values in that interval. For example, Figure 1 is a histogram of the array *lst* = [1, 15, 20, 1, 3, 11, 6, 5, 10, 13, 20, 14, 24]. In the figure, the 13 data values are split into five intervals or categories: [0, 5), [5, 10), [10, 15), [15, 20), [20, 25). The **interval notation**, such as [10, 15), is the set of numbers between the endpoints 10 and 15, including the value with the square bracket (10) but not the value with the parenthesis (15). Thus, for a number x in [10, 15), we have $10 \le x < 15$. Because four data values (10, 11, 13, 14) appear in this interval, the height of that bar is 4.

Definition A **histogram** of a data set is a bar chart where the base of each bar is an interval of data values and the height of this bar is the number of data values in that interval.

The *MATLAB* command ***hist*** produces a histogram of a list of numbers. The following code assigns a value to *lst* and displays its histogram with a default of 10 intervals or categories:

```
lst = [1, 15, 20, 1, 3, 11, 6, 5, 10, 13, 20, 14, 24];
hist(lst)
```

An optional second argument specifies the number of categories. Thus, the following command produces a histogram similar to that in Figure 1 with five intervals:

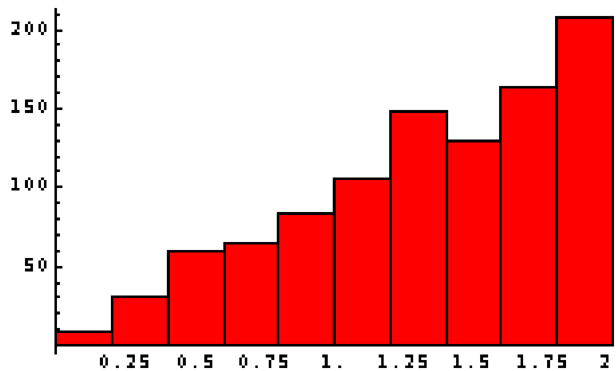
```
hist(lst, 5)
```

Figure 2 displays a histogram of an array, *tbl*, of 1000 random values from 0 to 2. (The table values only serve as data for graphing; each table entry is 2 times the maximum of two random numbers.) The commands to generate the table and produce the histogram are as follows:

```
tbl = 2 * max(rand(1, 1000), rand(1, 1000));
hist(tbl)
```

MATLAB uses 10 intervals, so each interval length is 0.25 (see Figure 2).

Figure 1 Histogram with 10 categories



Quick Review Question 8

a. Generate a table, *sinTbl*, of 1000 values of the sine of a random floating point number between 0 and π.
b. Display a histogram of *sinTbl*.
c. Display a histogram of *sinTbl* with 10 categories.
d. Give the interval for the last category.
e. Approximate the number of values in this category.

Maximum, Minimum, and Length

The *MATLAB* function ***max*** returns the maximum of a vector argument. Thus, the form of a call to the function can be as follows:

```
max([x1, x2, ... ])
```

In this case, the function returns the numerically largest of *x1*, *x2*, ... For example, the following invocation with four arguments returns the maximum argument, 7.

```
max([2, -4, 7, 3])
```

Another form of a call to the function, which follows, has a two-dimensional array of numbers as argument:

```
max([x11, x12,..., x1m; x21, x22,..., x2m; ...; xn1, xn2,..., xnm])
```

The result is a vector of the maximum in each column. The segment below generates a 3-by-3 array of numbers and returns the maximum values (3, 2, 5) in the columns:

```
lst = [-1, 2, -1; 0, 1, 5; 3, 2, 1];
max(lst)
```

To obtain the maximum (5) in the entire two-dimensional array, we take the maximum of this value, or the maximum of the maximum, as follows:

```
max(max(lst))
```

In a similar fashion, the *MATLAB* function ***min*** returns the minimum of a vector of numeric arguments; or in the case of a two-dimensional array, *min* returns a vector a minimum column values.

When we do not know the length of a vector, we can employ the ***length*** function, such as follows:

```
length(lst)
```

If *lst* were a matrix, this command would return its number of rows.

Quick Review Question 9 Start a new *Script*. In opening comments, have "MATLAB Tutorial 5 Answers" and your name. Save the file under the name *MATLABTutorial5Ans.m.* In the file, preface this and all subsequent Quick Review Questions with a comment that has "%% QRQ" and the question number, such as follows:

```
%% QRQ 1 a
```

a. Write a segment to generate a list, *lst*, of 100 random floating point numbers between 0 and 1 and to display the maximum and minimum in the list.

b. Write a segment to generate and display a vector of zeros, *z*, where the number of elements is a random integer between 1 and 20, and to return the number of elements in *z*.

Animation

One interesting use of a *for* loop is to draw several plots for an animation. In making the graphics, plots should have the same *axis* so that axes appear fixed during the animation. The segment below generates a sequence of plots of 0.5 sin(x), sin(x), 1.5 sin(x), ..., 5.0sin(x) with x varying from 0 to 2π to illustrate the impact the coefficient has on amplitude. However, for an effective demonstration, each graph must have the same display area, here from -5 to 5 on the y-axis.

```
x = 0:0.1:2*pi;
for i = 1:10
   y = 0.5 * i * sin(x);
   plot(x,y,'-');
   axis([0 2*pi -5 5]);
end
```

However, the animation can be fairly slow and jerky as *MATLAB* generates the animation one frame at a time.

Alternatively, we can store the frames in a vector and later replay the sequence as a movie. Immediately after generating the i-th frame of the animation, we save the graph in a vector using ***getframe***, as follows

```
M(i) = getframe;
```

After creating and storing all the frames, we show the animation with the ***movie*** command, which has the vector, here M, and the number of times to show the movie as arguments. Thus, the following command displays the animation 10 times:

```
movie(M, 10)
```

To display the animation one time with 5 **frames per second**, include a third parameter, such as follows:

```
movie(M, 1, 5)
```

The adjusted sequence to generate and play a movie twice with 4 frames per second follows:

```
x = 0:0.1:2*pi;
for i = 1:10
   y = 0.5 * i * sin(x);
   plot(x,y,'-');
   axis([0 2*pi -5, 5]);
   M(i) = getframe;
end
movie(M, 2, 4)
```

Quick Review Question 10 The first two statements of following segment generate 10 random *x*- and *y*-coordinates between 0 and 1. We then plot the points as fairly large blue dots in a 1-by-1 rectangle. Adjust the code to generate 10 plots. The first graphics only displays the first point; the second graphics shows the first two points; and so forth. After execution, animate to show the points appearing one at a time. Why do we want to specify the plot range to be the same for each graphic?

```
xLst = rand(1, 10);
yLst = rand(1, 10);
plot(xLst, yLst, 'ob', 'MarkerFaceColor', 'b')
axis([0 1 0 1])
```

Quick Review Question 11 The function *f* below is the logistic function for constrained growth, where the initial population is 20, the carrying capacity is 1000, and the rate of change of the population is (1 + 0.2*i*) (see Module 2.3, "Constrained Growth"). We use the operators ./ and .* so that the function works with arrays of numbers, such as t = 0:0.1:3, as well as numbers. To see the effect of increasing the rate of change from (1 + 0.2(**1**)) = 1.2 to (1 + 0.2(**10**)) = 3.0, replace the assignment of 5 to *i* with the start of a *for* loop to generate 10 plots of *f*(*t*, *i*), where *i* varies from 1 to 10. Create a movie of the animation that plays five times.

```
clear('i', 'f', 't');
f = @(t, i) (1000*20)./((1000 - 20).*exp(-(1+0.2*i).*t) + 20);
t = 0:0.1:3;
i = 5;  % replace with for loop
plot(t, f(t, i));
axis([0 3 0 1000]);
```